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Application of Trajectory Sensitivity for the Evaluation of the Effect of TCSC Placement on Transient Stability

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Abstract

This paper investigates the use of trajectory sensitivity analysis (TSA) technique for assessing the transient stability of a power system at various operating conditions. The effect of placement of a thyristor controlled series compensator (TCSC) as well as the influence of the change in firing angle is also discussed. The TCSC is modeled by a variable capacitor, the value of which changes with the firing angle. The systems studied are the WSCC 3-machine, 9-bus system and IEEE 16 machine 68 bus system. The results using TSA are validated by PSCAD/EMTDC simulation.

KEYWORDS: trajectory sensitivity analysis, contingency, TCSC

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I. INTRODUCTION

The use of FACTS controllers can be helpful in improving the efficiency of power system operation. Improved tools for assessing available stability margin of a system are required due to the open access nature of the deregulated system. This paper demonstrates that the Trajectory Sensitivity Analysis (TSA) can be a viable option. The technique can also be used for determining the suitable location of FACTS devices in a large interconnected network.

Transient energy function (TEF) method is a standard tool used for dynamic security assessment. But the use of TEF becomes too complex when detailed model of the system is considered such as systems with FACTS devices which are described by differential-algebraic equations (DAE) with a large number of algebraic constraints. The TSA is proposed in [1] and later extended to hybrid systems (i.e. DAE plus discrete event driven system) in [2]. A discussion about how the problems of TEF method can be overcome by TSA methods have been discussed in [3]. The application of TSA in dynamic security assessment is studied in detail in [3,4]. Besides computing critical values of parameters, the technique has been used for rescheduling of generation as part of a dynamic security assessment (DSA) scheme. Application of TSA in transmission system protection to detect unstable power swings and electrical centers is described in [5].

FACTS controllers like TCSC can be placed in one of the lines of a power system with suitable control scheme to improve the transient stability condition of the system [6]. A method for reducing the number of trajectory sensitivity calculations to get the most effective control is described in [7].

In this paper, the TSA technique is applied with the nominal trajectory being the system with a fault, which is subsequently cleared. Systems with load modeled as i) constant impedance and ii) constant PQ are investigated. Fault in one of the lines is simulated and the fault clearing time is chosen as the parameter. Transient stability margin (TSM) is computed in all the cases. TSM is a metric computed from the trajectory sensitivities.

A TCSC is then placed in one of the lines for compensation and the TSM is computed. The TCSC is represented by a fundamental frequency lumped reactance model that varies with the change in the firing angle. The TSM varies with the placement of TCSC at different lines of the system and with various amounts of compensation. The study is useful for the planner as well as operation personnel. Some of the results obtained are validated through PSCAD/EMTDC simulation studies.

II. TRAJECTORY SENSITIVITY ANALYSIS IN MULTIMACHINE POWER SYSTEMS

A. Computation of Trajectory Sensitivity

Multi-machine power systems are generally modeled by DAEs. Consider such a system given by

$$\begin{aligned} \dot{x} &= f(x, y, \alpha), \quad x(t_0) = x_0 \\ 0 &= g(x, y, \alpha), \quad y(t_0) = y_0 \end{aligned} \tag{1}$$

where $x \in \Re^n$ is the state vector, y is a vector of algebraic variables and α is a vector of system parameters. The trajectory sensitivity of such a system can be computed with respect to changes in system parameters (α), initial conditions (x_0,y_0) or order of the system (n) [8,9]. However in this paper we consider sensitivity with respect to system parameters only.

The parameter α is perturbed from the nominal value of α_0 and the equations for trajectory sensitivity can be found as [1]

$$\begin{aligned} \dot{w}_1 &= \left[\frac{\partial f}{\partial x}\right] w_1 + \left[\frac{\partial f}{\partial y}\right] w_2 + \left[\frac{\partial f}{\partial \alpha}\right], \quad w_1(t_0) = 0 \\ 0 &= \left[\frac{\partial g}{\partial x}\right] w_1 + \left[\frac{\partial g}{\partial y}\right] w_2 + \left[\frac{\partial g}{\partial \alpha}\right], \quad w_2(t_0) = 0 \end{aligned} \tag{2}$$

where $w_1 = \partial x/\partial\alpha$ and $w_2 = \partial y/\partial\alpha$ are the sensitivities. Then Solving (1) and (2) simultaneously, we get x, y and the sensitivities w_1 and w_2.

B. Numerical Evaluation: Alternative to Reduce Computation

The sensitivities $\partial x/\partial\alpha$ and $\partial y/\partial\alpha$ are calculated by solving an extra set of differential and algebraic equations. This can be substituted by a simpler method where sensitivities are calculated numerically. To explain this method, let us choose only one parameter, i.e., α becomes a scalar and the sensitivities with respect to it are studied. Two values of α are chosen (say α_1 and α_2). The corresponding state vectors x_1 and x_2 respectively are then computed. Now the sensitivity at α_1 is defined as

$$Sens = \frac{x_2 - x_1}{\alpha_2 - \alpha_1} = \frac{\Delta x}{\Delta \alpha} \tag{3}$$

If $\Delta\alpha$ is small, the numerical sensitivity is expected to be very close to the analytically calculated trajectory sensitivity value.

C. Multi-Machine Power System Model

Consider an n-bus m-machine system with the machines represented by the classical model. The buses are numbered 1 to n and the internal nodes of the machines are represented by (n+1) to (n+m). In the classical model each machine is represented by a voltage of constant magnitude behind the transient reactance of the machine. The rotor dynamics is considered by taking the swing equations as

$$\frac{d\theta_i}{dt} = \omega_s \Delta\omega_{r_i}, \quad i = n+1, \ldots, n+m \tag{4}$$

$$2H_i \frac{d\Delta\omega_{r_i}}{dt} = P_{m_i} - \sum_{j=1}^{n+m} |V_i|.|V_j| [G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)] - K_{D_i}\Delta\omega_{r_i}, \quad i = n+1, \ldots, n+m \tag{5}$$

where, $\Delta\omega_r$ is the per unit speed deviation, H is the inertia constant, ω_s is the synchronous speed, m is the number of machines, P_m the mechanical power input in per unit. It is to be noted that

- V_i (i = 1… n) are the bus voltages in per unit and θ_i (i = 1… n) are their angles in radian.
- θ_i (i = n + 1… n + m) are the angular positions of the rotors (i.e., δ_i's) and V_i (i = n +1… n+m) are the constant magnitude internal voltages of the machines.

The dynamics of the network and the stator windings are neglected and the network is represented by a set of algebraic equations as

$$P_{L_i} = \sum_{j=1}^{n+m} |V_i||V_j| [G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)], \quad i = 1, \ldots, n \tag{6}$$

$$Q_{L_i} = \sum_{j=1}^{n+m} |V_i||V_j| [G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)], \quad i = 1, \ldots, n \tag{7}$$

where G_{ij} and B_{ij} are the network transfer conductance and admittance respectively. These are obtained from the augmented Y_{BUS} matrix where the admittance corresponding to the transient reactance of the machines are included along with normal Y_{BUS}. P_{Li} and Q_{Li} are the real and reactive powers load at the i^{th} bus respectively. This representation of a power system is known as the structure-preserving model [10, 11].

D. Quantification of TS and Its Implication

A new term η (ETA), introduced in [3] to quantify the information content of TS, is used here. Consider the m-machine power system, the state vector of which contains the rotor angle δ and the relative rotor speed $\Delta\omega$ of each machine. Then the sensitivity norm is calculated as

$$S_N = \sqrt{\sum_{i=1}^{m}\left[\left(\frac{\partial\delta_i}{\partial\alpha} - \frac{\partial\delta_j}{\partial\alpha}\right)^2 + \left(\frac{\partial\Delta\omega_{r_i}}{\partial\alpha}\right)^2\right]} \tag{8}$$

where the j^{th} machine is taken as the reference. Then η is defined as the inverse of the maximum of S_N.

$$\eta = \frac{1}{\max(S_N)} \tag{9}$$

As the system moves towards instability, the oscillation in TS will be more resulting in larger values of S_N. This will result in the smaller values of η. Ideally η should be zero at the point of instability. Therefore the value of η gives us an indication of distance from instability. In this paper η is used for assessing the relative stability conditions of the system with different values of fault clearing time, system load and firing angle of TCSC.

Consider a fault in one of the lines of the system. The post-fault conditions are studied by continuously increasing the fault clearing time (t_{cl}). The system states will oscillate more and take longer time to settle as t_{cl} is increased. The sensitivities of the state variables will also exhibit large oscillations for increasing t_{cl}. These oscillations will become unbounded as t_{cl} exceeds the critical clearing time. Thus large peaks in trajectory sensitivity (TS) clearly indicate the proximity of the parameter to the critical value beyond which the system becomes unstable. It has been shown in [9] that the variations of TS are much prominent than the state trajectory itself as the system approaches instability. Therefore TSA can be used as a transient stability indicator in a power system.

E. Simulation of fault

A symmetrical fault is simulated in one of the lines at a time. The simulation is done in three phases:

1. The pre-fault system is run for a small time (say 0.1 second).
2. The fault is applied at one end of the line. Simulation of this faulted condition continues till the line is disconnected from the buses at both the ends of the faulted line after a time t_{cl}. The time gap between the tripping of breakers at the two ends is negligible compared to the clearing time.

Hence the disconnection of the line at the two ends can be considered simultaneous.

3. Next is the post-fault system simulation where the faulty line is totally disconnected from the system. Simulation is carried out for a longer time (say 10seconds) to observe the nature of the transients.

This completes the simulation of the system for one value of the parameter which is fault-clearing time (t_{cl}) here. For finding TS, step 2 and 3 is repeated with a slightly higher value of t_{cl}. The amount of perturbation in t_{cl} used in this study is 0.005 s. The η, which is a measure of TS at tcl, is then computed using (8-9).

Two separate simulations are done, one with load modeled as constant PQ and the other as constant impedance. For the simulation with constant PQ model the loads are converted to constant impedance during the fault period for better convergence. Once the fault is removed, load is again converted into constant PQ.

F. Modeling of TCSC

The TCSC model is given in Fig. 1. The overall reactance X_C of the TCSC is given in terms of the firing angle α as [13]

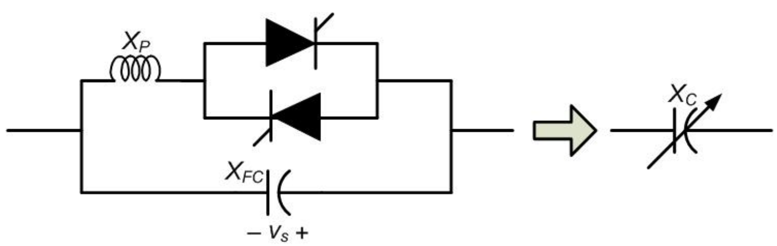

Fig 1 TCSC circuit and its equivalent

$$X_C = \beta_1\left(X_{FC} + \beta_2\right) - \beta_4\beta_5 - X_{FC} \tag{10}$$

where

$$\beta_1 = \frac{2(\pi - \alpha) + \sin 2(\pi - \alpha)}{\pi},\ \beta_2 = \frac{X_{FC}X_P}{X_{FC} - X_P},\ \beta_3 = \sqrt{\frac{X_{FC}}{X_P}}$$

$$\beta_4 = \beta_3 \tan\left[\beta_3(\pi - \alpha)\right] - \tan(\pi - \alpha), \quad \beta_5 = \frac{4\beta_2^2\cos^2(\pi - \alpha)}{\pi X_P}$$

Here C is the fundamental frequency capacitance of the TCSC and is equal to $1/(\omega_s X_C)$. It is to be noted that in this paper the TCSC is operated only in the capacitive mode. Therefore the value of firing angle α is chosen to be between 140° and 180°. TCSC is placed in one of the lines of the network. The capacitive reactance X_{FC} of the TCSC is chosen to be half the reactance of the line (in which it is placed) and the TCR reactance X_P is chosen to be one third of $X_{FC.}$ The net reactance of the line is calculated as the difference between the line reactance and the TCSC reactance. The percentage compensation obtained at 160° and 145° are 53.10 and 74.54 respectively. The boost factor, defined as the ratio of X_C to X_{FC}, for these two α are 1.0619 and 1.4907 respectively.

III. APPLICATION OF TSA IN A 9 BUS SYSTEM

First, we consider the WSCC 3-machine, 9-bus system, shown in Fig. 2 [10].

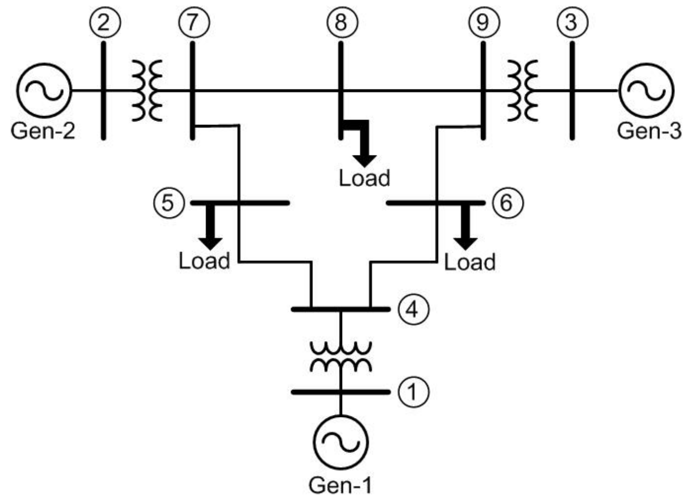

Fig. 2 Single line diagram of the WSCC 9 bus system

A. System condition on application of fault without TCSC

Initially the TCSC is not included and the system is subjected to a fault in one of its lines. Fault clearing time (t_{cl}) is taken as 0.1 s, which is actually 6 cycles for a 60 Hz system and the TS, and η are computed. Next, t_{cl} is increased in steps to observe the change in TS in the post fault phase as represented by η. The results for the system with constant PQ load and constant impedance load are given in Table I.

It can be observed from the table that the value of η decreases with increase in clearing time for all the fault locations. As per the discussion of Sec II (D), this indicates deterioration in stability. It is in accordance with the expectation that the stability condition of a power system would deteriorate as the fault duration is increased. This shows the effectiveness of TS and η as a stability margin indicator. Next, the value of t_{cl} is increased to find the critical clearing time (the value of t_{cl} beyond which the system becomes unstable). Convergence problems are encountered at larger clearing time while doing simulation with load as constant PQ; hence the load model is converted to constant impedance in those cases. The values of critical clearing time (t_{cr}) for different fault locations are given in the last column of Table I. It can be seen that the values of t_{cr} are higher in the cases with higher values of η and vice versa. For example, the value of η is highest for a fault in line 4-6, indicating a comparatively higher stability margin. The value of t_{cr} is also highest in this case. Similarly, the value of η is lowest in case of a fault in line 7-8, thus indicating a system more near to instability. The value of t_{cr} is also lowest in this case. The results are verified by PSCAD/EMTDC simulation. Plots of relative machine angles are shown in Figs. 3(a) and 3(b). They show the responses of relative rotor angle δ_{31} (between generator 3 and generator 1) when a fault is applied in line 7-8 and 6-9 respectively. Duration of fault in each case is same with t_{cl} of 0.15s.

TABLE I: VARIATION OF η WITH t_{cl}, 9-BUS SYSTEM

Fault applied in line	Constant PQ Load		Constant Impedance Load		Critical clearing time (sec)
	Fault clearing time		Fault clearing time		
	0.10s	0.15s	0.10s	0.15s	
4-6	0.2318	0.2035	0.2313	0.2034	0.605
5-7	0.2242	0.1979	0.2260	0.1998	0.440
6-9	0.2310	0.2029	0.2316	0.2038	0.540
7-8	0.1015	0.0370	0.1640	0.0866	0.330
8-9	0.1640	0.0793	0.1648	0.1140	0.360

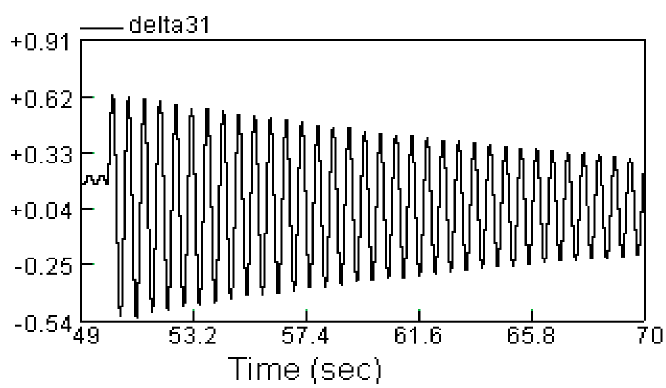

Fig 3(a) Response of relative rotor angle δ_{31} for fault in line 7-8

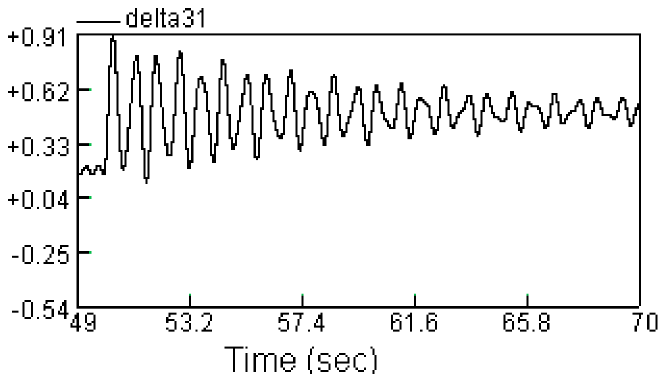

Fig 3(b) Response of relative rotor angle δ_{31} for fault in line 6-9

B. Changes in Stability Condition with Inclusion of TCSC

With fault in one of the lines of the 9-bus system, the TCSC is placed in one of the other lines to study the effect on transient stability condition. For a given t_{cl} the firing angle α is varied to apply different amounts of compensation. This study is carried out with the fault in different lines of the 9-bus system. Table II shows the results for t_{cl} =0.15s and $\alpha = 160^o$ and 145^o. The values of η for the system without TCSC are termed as base η. These values are given in the first column of the Table (along with the faulted line). The same analysis is carried out with the load modeled as constant impedance and the results obtained are also given in Table II.

Results in Table II indicate that for fault in a particular line, the improvement of system stability on introduction of TCSC is very much dependent on the choice of suitable location of the TCSC. For example, when the fault is in line 4-6, the highest value of η and hence the maximum improvement of system stability is obtained for the TCSC placed in line 6-9 (α=145^o) whereas for fault in line 8-9, TCSC in line 5-7 gives the maximum benefit. It can also be noted that placement of TCSC in some of the lines may even lead to a deterioration of the stability condition of the system. One such example is the placement of TCSC in line 4-6 when fault is in line 6-9. As we reduce the firing angle α of TCSC from 160^o to 145^o, amount of compensation increases. However, this is not always beneficial from the system stability viewpoint. For example, for fault in line 7-8 and TCSC in line 6-9, the stability condition deteriorates when firing angle is decreased. Therefore, in addition to the placement, TS also helps in deciding the suitable firing angle for the TCSC operation. The simulation results to verify the change in system stability condition with placement of TCSC are shown in Figs. 4(a) and 4(b). In both these cases, a fault of duration 0.15 second in line 6-9 is simulated. Fig. 4(a) shows the response of relative machine angles $\delta_{3\text{-}1}$ when the TCSC is placed in line 4-6 whereas Fig. 4(b) shows $\delta_{3\text{-}1}$ for TCSC placed in line

5-7. The firing angle of TCSC is 160° in both the cases. It is clear that the maximum peak of the response is much lower in case of TCSC in line 5-7, which indicates a higher transient stability margin. This is in accordance with the higher value of η for TCSC in line 5-7 than in line 4-6 in the corresponding case as given in Table II.

TABLE II: VARIATION OF η WITH TCSC

Load type	Fault in line, Base η	α (deg)	η Values for TCSC placed in line				
			4-6	5-7	6-9	7-8	8-9
Constant PQ	4-6, 0.2035	160	--	**0.2425**	0.2372	0.2054	0.2033
		145	--	0.2678	**0.3099**	0.2065	0.2027
	6-9, 0.2029	160	0.1539	**0.2427**	--	0.2047	0.2029
		145	0.0999	**0.2683**	--	0.2057	0.2023
	7-8, 0.0370	160	**0.0434**	0.0411	0.0378	--	0.0301
		145	**0.0443**	0.0399	0.0370	--	0.0280
	8-9, 0.0793	160	0.0772	**0.0894**	0.0783	0.0614	--
		145	0.0734	**0.1004**	0.0726	0.0546	--
Constant Impedance	4-6, 0.2034	160	--	**0.2425**	0.2364	0.2054	0.2033
		145	--	0.2678	**0.3076**	0.2064	0.2027
	6-9, 0.2038	160	0.1556	**0.2429**	--	0.2057	0.2036
		145	0.1286	**0.2682**	--	0.2067	0.2030
	7-8, 0.0866	160	0.0896	0.0885	**0.0925**	--	0.0657
		145	0.0876	**0.0921**	0.0898	--	0.0589
	8-9, 0.1140	160	0.1117	**0.1255**	0.1146	0.0886	--
		145	0.1071	**0.1366**	0.1072	0.0787	--

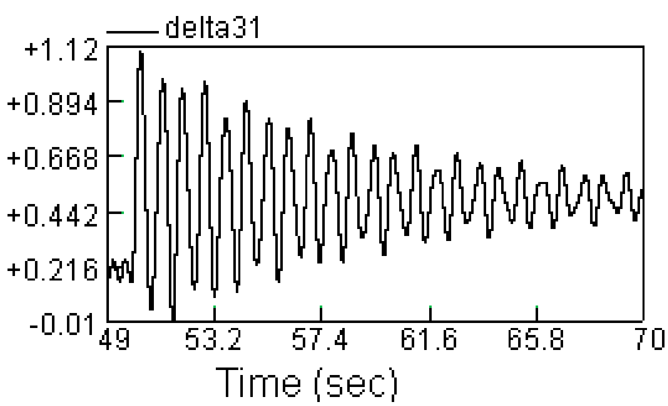

Fig. 4(a) Relative rotor angle $\delta_{3\text{-}1}$ for a fault in line 6-9 with TCSC in line 4-6

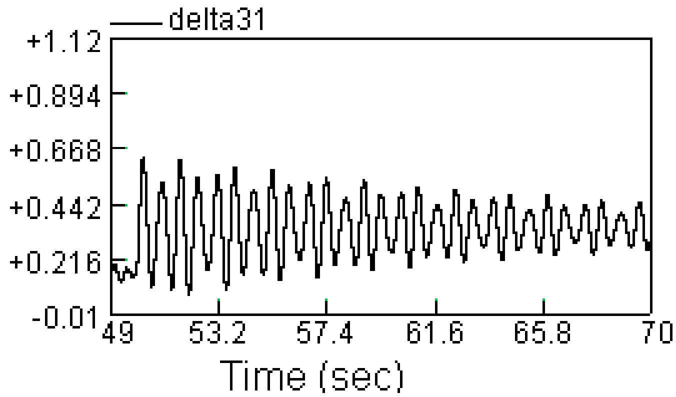

Fig. 4(b) Relative rotor angle $\delta_{3\text{-}1}$ for a fault in line 6-9 with TCSC in line 5-7

Comparison between results for system with constant PQ load and constant impedance load shows that the results are similar in most of the cases. The values of η are not exactly same but the suitable placement position and firing angle found are same in most of the cases. However, it can be noted from the Tables I and II that the values of η are higher in all the cases with constant impedance load. This means that the modeling of load as constant impedance gives a comparatively optimistic result. In the remaining part of this paper load is considered as constant PQ, so that the transient stability margin estimation is more on the conservative side giving a safer margin.

We can see that the effectiveness of the TCSC varies with the fault location, resulting in different preferred placement locations. Therefore, it is difficult to identify a single location for the FACTS device such that when it is placed in that particular location, it will most effectively counter instability due to fault in any corner of the system. However, if one or a few critical fault locations are known where the fault is most likely to occur and/or where the fault makes the system most vulnerable to instability, then one can use this TSA based method to find out the best possible location for the device. In practical systems, the operators and planners do have this sort of information (regarding fault prone lines) and hence this method can be very useful.

C. Effect of Load increase and Generation Rescheduling

We next discuss what happens when the loading conditions and generations of the three generators are varied. These variations result in changes in the power flow through different lines. The trajectory sensitivities with respect to t_{cl} and η are computed under these changed conditions. Load at all the buses are increased simultaneously by the same percentage (15%) and one generator picks the extra load at a time. The results for variation of η are given in Tables III (A) (generator

1 picks extra load) and III (B) (generator 2 picks extra load). The fault clearing time is 0.10 s and firing angle is 145^o in all the cases. The base η given in the first column of the tables (along with the faulted line number) is the η value for fault in that line with no TCSC in the system. The last column contains the η value for fault in the corresponding line when the load is 100% and no TCSC is placed. The abbreviation 'DNC' in the tables mean "Do Not Converge", i.e., at those loading conditions the Newton-Raphson iteration used in SI simulation does not converge.

TABLE III(A): VARIATION OF η WITH 15% LOAD INCREASE, EXTRA LOAD PICKED BY GENERATOR 1

Faulted line, base η	TCSC placed in line					η at nominal load
	4-6	5-7	6-9	7-8	8-9	
4-6, 0.3069	--	0.4098	**0.5118**	0.3135	0.3044	0.2318
5-7, 0.3019	0.3202	--	**0.3877**	0.3053	0.3088	0.2242
6-9, 0.3092	0.2010	**0.4149**	--	0.3148	0.3070	0.2310
7-8, 0.0811	**0.1261**	0.1049	0.0869	--	0.0900	0.1015
8-9, 0.1812	0.1800	**0.1867**	0.1843	0.1673	--	0.1640

TABLE III(B): VARIATION OF η WITH 15% LOAD INCREASE, EXTRA LOAD PICKED BY GENERATOR 2

Faulted line, base η	TCSC placed in line					η at nominal load
	4-6	5-7	6-9	7-8	8-9	
4-6, 0.1885	--	**0.2576**	0.2407	0.1895	0.1844	0.2318
5-7, D.N.C.	DNC	--	**0.2030**	DNC	DNC	0.2242
6-9, 0.1857	0.0560	**0.2588**	--	0.1861	0.1827	0.2310
7-8, D.N.C	DNC	**0.0482**	DNC	--	DNC	0.1015
8-9, 0.0655	0.0579	**0.1263**	0.0677	0.0501	--	0.1640

For the case of fault at higher loading conditions, the results show that the system stability margin is better if the extra load is picked by generator 1. For example for a fault in line 4-6, the η value at nominal load is 0.2318 and it decreases to 0.1885 when load is increased by 15% and the extra load is supplied by generator 2. On the contrary, the η value increases to 0.3069 when the extra load is supplied by generator 1. Therefore, if the load is expected to be more than 100% and the system does not have any TCSC then it will be better to supply the extra load from generator 1 and not generator 2 for system safety. Another important aspect to be observed is the change in preferable positions for TCSC placement under overloaded conditions. For example for a fault in the line 4-6 under normal loading condition, the most suitable placement (i.e. where value of η is maximum) is line 6-9. But with 15% overload and generator 2 supplying that extra load, line 5-7 becomes the most suitable placement for a fault in line 4-6. Thus TSA and η can be useful to determine the preferable generator loading and suitable placement positions of the TCSC at various loading and generation combinations.

IV. ANALYSIS OF 68 BUS SYSTEM USING TSA

The single line diagram of the 68-bus system is shown in Fig 5. It consists of 16 generators, 68 buses and 86 lines [14]. The generators are represented by the classical model as before. System damping values are taken according to the data given in [14]. The loads are considered as constant PQ.

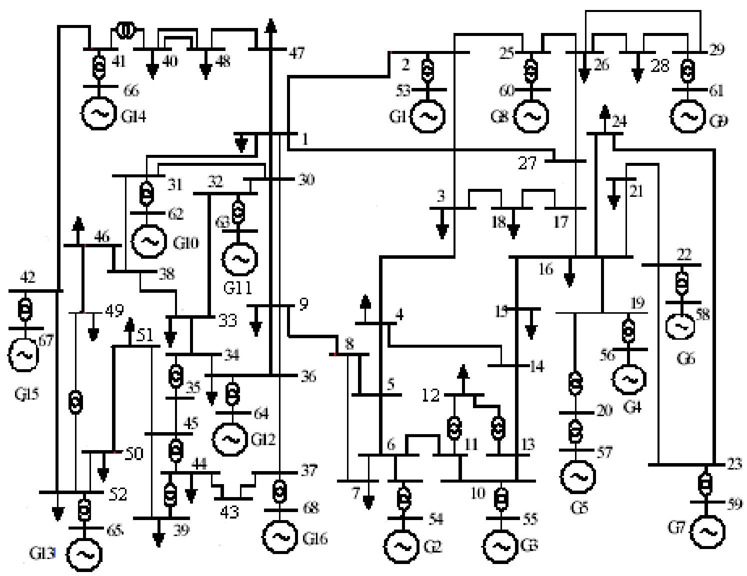

Fig. 5 Single line diagram of the 16 machine 68 bus system

A. System condition on application of fault without TCSC

A three-phase fault with line switching is applied in one of the lines of the 68-bus system. The procedure is same as in Section II (E). At first, t_{cl} is 0.05s and then it is increased in steps. Fault is simulated in different lines at varied location in the system (one at a time) by the above procedure. The results for t_{cl} equal to 0.05s, 0.1 are given in Table IV. It is quite apparent from the results that the η values decrease (i.e. sensitivity values increase) with the increase in fault clearing time for all the lines as expected. This is in accordance with the fact that with increase in t_{cl} the system moves more near to transient instability. However, the reduction in system stability margin is different for fault in different lines. The values of critical clearing time (t_{cr}) are also given in the last column of the Table. As we know, t_{cr} is also an indication of stability margin of the system. It can be observed from the Table that the t_{cr} values are consistent with the η values. The t_{cr} is small in cases with small η and vice versa. For example, for a fault in line 16-15, η is found to be 0.0052, which is very near to zero thus indicating close proximity to instability. The corresponding t_{cr} is also very low (0.125s). Similarly, A fault in line 26-25 also reduces stability margin considerably as indicated by low η values of 0.0220. The t_{cr} is 0.14s in this case. On the other hand, fault in line 1-47 or 30-9 damages system stability condition by much less amount as indicated by the higher values of η (0.1074 and 0.0789 respectively). The corresponding values of t_{cr} are comparatively high (0.395s and 0.335s respectively).

TABLE IV: VARIATION OF η WITH t_{cl}, 68-BUS SYSTEM

Fault applied in line	Fault clearing time		Critical clearing time
	0.05s	0.10s	
3-2	0.0744	0.0221	0.190s
5-6	0.0899	0.0380	0.200s
11-10	0.0975	0.0470	0.220s
16-15	0.0525	0.0052	0.125s
26-25	0.0790	0.0220	0.140s
30-9	0.1260	0.0789	0.335s
1-47	0.1450	0.1074	0.395s

These results can be verified from the plots of relative rotor angles. Fig. 6 (a) shows the three dimensional plots of rotor angles of machines 1 to 9 relative to that of machine 16 (taken as the reference) for a fault in line 3-2. The same angles for a fault in line 1-47 are shown in Fig. 6 (b). The plots of relative rotor angle δ_{2-16} for faults in line 3-2 and line 1-47 are shown in Fig. 7 for comparison. It can be seen from these figures that the oscillations in relative rotor angles are much more in case of fault in line 3-2 indicating a less stable system.

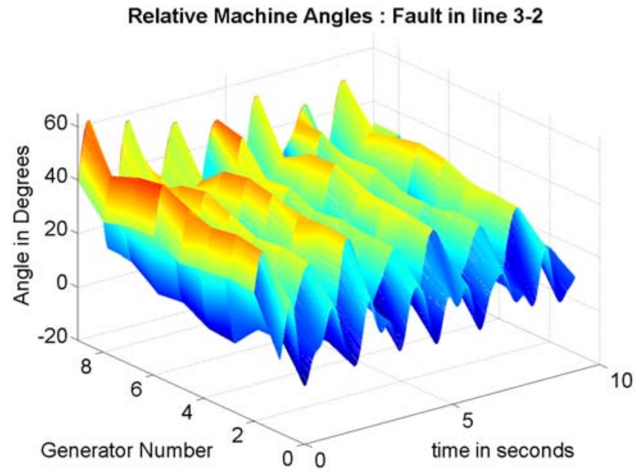

Fig. 6(a) Responses of relative rotor angles for a fault in line 3-2

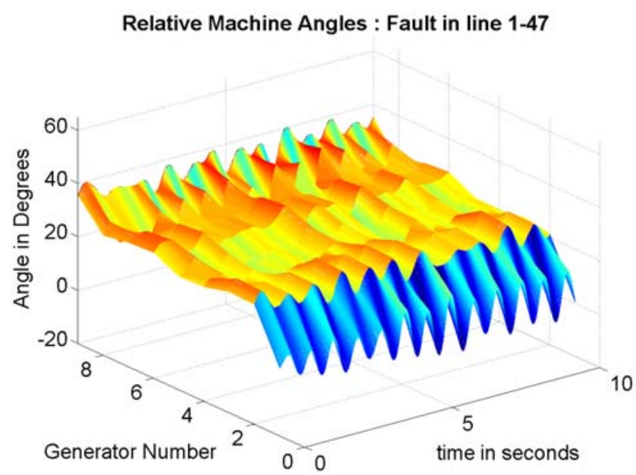

Fig. 6(b) Responses of relative rotor angles for a fault in line 1-47

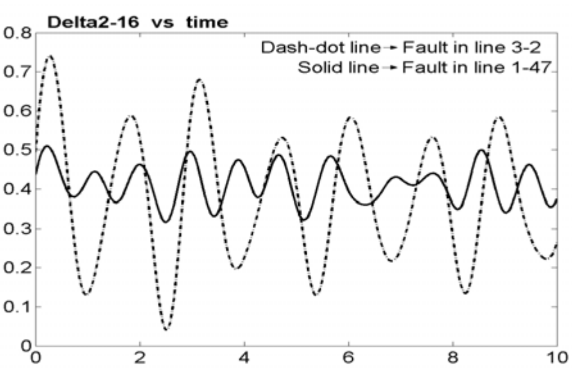

Fig 7 Response of relative rotor angle $\delta_{2\text{-}16}$

B. Changes in Stability Condition on Application of TCSC

System gets close to transient instability in five of the eight cases (fault in line 3-2,5-6,11-10,16-15,26-25) studied in the previous section. So, TCSC is now employed for these cases. However, it is to be noted that this work does not aim to find out the best possible location for TCSC placement out of all the 86 lines for each of the cases. So, in the absence of any analytical criteria, 10 lines are chosen arbitrarily for each of the cases and the TCSC is placed in those lines, one at a time, and the effect is studied. The results are given in Table V (A)-V (E). The fault duration is taken as 0.10s in each of the cases and the firing angle of the TCSC is taken to be 160^{o}. Comparison of the η values after TCSC placement with the base η values (i.e. the value without TCSC) gives a clear picture of the effect of the TCSC placement in different lines. Similar to the 9 bus system results, it is found that putting the TCSC in some of the lines help in system stability whereas for the TCSC put in some other lines actually damage the system stability. That highlights the importance of tools like TSA for the assessment of stability.

It can be seen from the results that with fault in line 3-2, placement of TCSC in line 8-9 improves the system stability as indicated by the increase in η value from 0.0221 (base case) to 0.406. On the other hand, placement of TCSC in line 3-4 drags the system more near to instability as indicated by the fall in the value of η to 0.0166. Similarly, for fault in lines 5-6, 11-10, 16-15 and 26-25 system stability condition improves most with the TCSC in lines 6-7, 4-14, 16-17 and 26-27 respectively out of the 10 lines considered in each case. Another important point is that placing TCSC in the lines adjacent to the faulty line is not necessarily beneficial. For example, in case of a fault in line 11-10, the stability margin deteriorates for TCSC in 6-11 - one of the adjacent lines whereas it improves for TCSC in 4-14, which is not in the vicinity of the faulty line. Similar is the case for fault in line 3-2 where 3-4 is an adjacent line whereas 8-9 is not that close. These results are supported by the plots of relative rotor angles for the system with and without TCSC as given in Fig 8 (a)-(b). These are for fault in line 3-2 of duration 0.1 second. The TCSC is in line 8-9 and firing angle is 160^{o}. Fig 8 (a) and (b) shows the responses of relative rotor angle $\delta_{2\text{-}16}$, and $\delta_{9\text{-}16}$ respectively.

It is clear from the figures that the oscillations reduce when TCSC is placed in line 8-9 indicating a more stable system. The oscillations reduce further when the firing angle of the TCSC is changed to 145°. Also the value of η increases to 0.0530 in this case. The plots of $\delta_{2\text{-}16}$ and $\delta_{9\text{-}16}$ for TCSC with $\alpha=160^{\circ}$ and $\alpha=145^{\circ}$ are given in Fig 9 (a)-(b).

TABLE V(A): EFFECT OF TCSC ON η, FAULT IN LINE 3-2, BASE η WITHOUT TCSC = 0.0221

TCSC in line	η	TCSC in line	η
17-27	0.0224	1-2	0.0295
25-26	0.0237	1-30	0.0227
2-25	0.0227	3-18	0.0180
8-9	**0.0406**	17-18	0.0200
3-4	0.0166	4-14	0.0222

TABLE V(B): EFFECT OF TCSC ON η, FAULT IN LINE 16-15, η WITHOUT TCSC = 0.0052

TCSC in line	η	TCSC in line	η
16-17	**0.0064**	13-14	0.0052
3-4	0.0061	10-11	0.0052
14-15	0.0053	3-18	0.0060
4-5	0.0053	17-18	0.0057
6-11	0.0053	4-14	0.0052

TABLE V(C): EFFECT OF TCSC ON η, FAULT IN LINE 26-25, η WITHOUT TCSC = 0.0220

TCSC in line	η	TCSC in line	η
26-27	**0.0291**	14-15	0.0212
17-27	0.0290	15-16	0.0215
2-25	0.0216	3-18	0.0224
2-3	0.0237	17-18	0.0221
4-5	0.0218	4-14	0.0217

TABLE V(D): EFFECT OF TCSC ON η, FAULT IN LINE 5-6, η WITHOUT TCSC = 0.0380

TCSC in line	η	TCSC in line	η
7-8	0.0397	10-13	0.0384
6-7	**0.0427**	10-11	0.0375
5-8	0.0374	3-18	0.0374
13-14	0.0386	17-18	0.0379
6-11	0.0367	4-14	0.0390

TABLE V(E): EFFECT OF TCSC ON η, FAULT IN LINE 11-10, η WITHOUT TCSC = 0.0470.

TCSC in line	η	TCSC in line	η
10-13	0.0457	12-13	0.0474
13-14	0.0426	16-17	0.0476
6-11	0.0387	3-18	0.0460
5-6	0.0461	17-18	0.0467
12-11	0.0471	**4-14**	**0.0492**

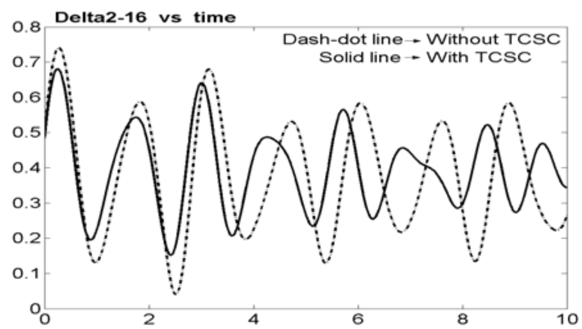

Fig 8(a) Response of $\delta_{2\text{-}16}$ with and without TCSC, fault in line 3-2

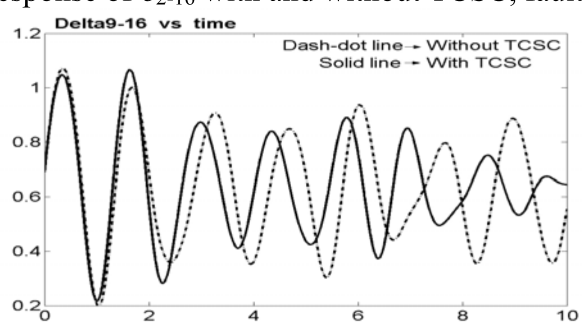

Fig 8(b) Response of $\delta_{9\text{-}16}$ with and without TCSC, fault in line 3-2

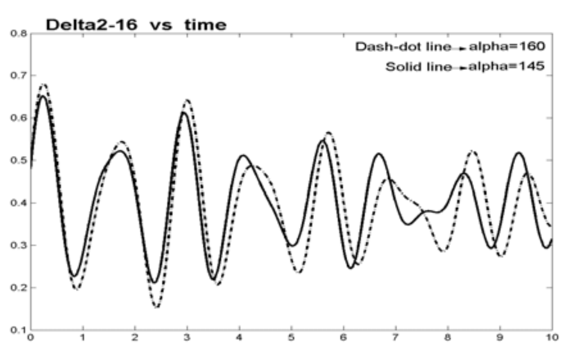

Fig 9(a) Response of $\delta_{2\text{-}16}$ with different TCSC firing angles

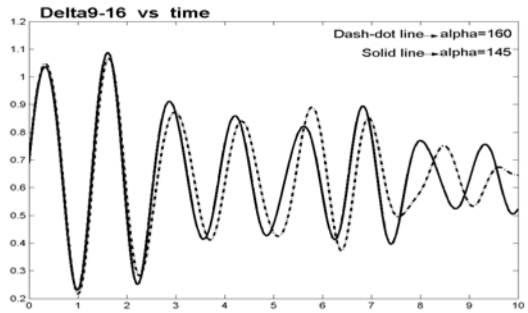

Fig 9(b) Response of $\delta_{9\text{-}16}$ with different TCSC firing angles

V. TSA WITH FLUX-DECAY MODEL OF GENERATORS

A more detailed model of the generators is considered in this section and it is shown that TSA can be applied there also.

A. Model of Generators

The synchronous machines are represented by the flux-decay model as shown in Fig 10 (a) [10]. A simplified static exciter model with one gain and one time constant is considered, as in Fig. 10 (b).

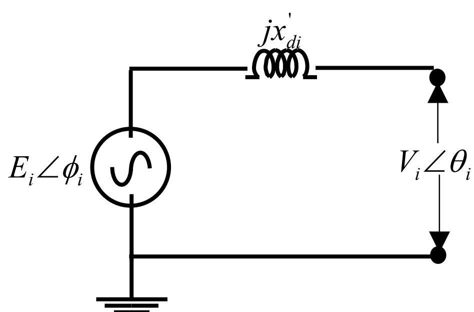

Fig.10 (a) Flux-decay model of generator

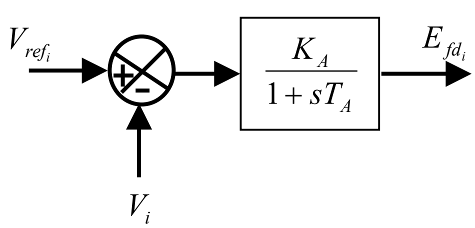

Fig.10 (b) Static exciter model

The internal voltage of the generator and its angle, $E_i\angle\phi_i$ are given by

$$E_i\angle\phi_i = \left[\left(1-\frac{x'_{d_i}}{x_{q_i}}\right)V_i \sin\left(\delta_i-\theta_i\right)+jE'_{q_i}\right]e^{j(\delta_i-\pi/2)} \tag{11}$$

The generator and exciter dynamics are described by the following equations [9,10]

$$\frac{d\delta_i}{dt}=\omega_s\Delta\omega_{r_i}, \quad i=1,\ldots,m \tag{12}$$

$$2H_i\frac{d\Delta\omega_{r_i}}{dt}=P_{m_i}-P_{e_i}-K_{D_i}\Delta\omega_{r_i}, \quad i=1,\ldots,m \tag{13}$$

$$T'_{do_i}\frac{dE'_{q_i}}{dt}=-\frac{x_{d_i}}{x'_{d_i}}E'_{q_i}+\left(\frac{x_{d_i}}{x'_{d_i}}-1\right)V_i\cos(\delta_i-\theta_i)+E_{fd_i}, \quad i=1,\ldots,m \tag{14}$$

$$T_{A_i}\frac{dE_{fd_i}}{dt}=-E_{fd_i}+\left(V_{ref_i}-V_i\right)K_{A_i}, \quad i=1,\ldots,m \tag{15}$$

$$P_{e_i}=E'_{q_i}V_i\sin\left(\delta_i-\theta_i\right)/x'_{d_i}+0.5\left(1/x_q-1/x'_{d_i}\right)V_i^2\sin 2\left(\delta_i-\theta_i\right), \quad i=1,\ldots,m \tag{16}$$

where, δ is the angular position of the rotor, ω_r is the rotor speed, $\Delta\omega_{ri}$ per unit speed deviation, ω_s is the synchronous speed, m is the number of machines, H is the inertia constant, K_D is the damping coefficient, P_m is the mechanical power input, P_e is the electrical power output, x_d and x_q are the d-axis and q- axis synchronous reactance, x'_d is the d-axis transient reactance, T'_{do} is the d-axis open circuit time constant, K_A and T_A are the gain and time constant of the exciter, V is the terminal voltage of the machine in per unit and θ is its angle. The network is represented by equations (6-7) as before.

B. Results of TSA and comparison with results for Classical Model

Trajectory sensitivities are computed as described in Sec. II-B. The sensitivity norm and η are also computed using equations (8-9). Results for a few cases of the 9-bus system and 68-bus system are given in Table VI (A) and Table VI (B) respectively. Load is considered constant PQ and firing angle of TCSC is taken as 160°.

TABLE VI(A)
VARIATION OF η WITH t_{cl} AND TCSC PLACEMENT, 9-BUS SYSTEM

Fault applied in line	Without TCSC		With TCSC				
	Fault clearing time		η Values for TCSC placed in line				
	0.10s	0.15s	4-6	**5-7**	6-9	7-8	8-9
4-6	0.1773	0.1493	--	**0.1728**	0.1435	0.1493	0.1502
6-9	0.1451	0.1387	0.1121	**0.1646**	--	0.1385	0.1421

TABLE VI(B)
VARIATION OF η WITH t_{cl} AND TCSC PLACEMENT, 68-BUS SYSTEM

Fault applied in line	Without TCSC		With TCSC
	Fault clearing time		Most Beneficial TCSC location
	0.05s	0.10s	
3-2	0.0705	0.0657	Line 8-9
16-15	0.0536	0.0417	Line16-17

In Table VI (A), the values of η are shown for increasing t_{cl} for two fault locations (one at a time) of the 9-bus system without TCSC. It can be seen that in case of a fault in line 4-6, η decrease from 0.1773 to 0.1493 and for a fault in line 6-9, η decreases from 0.1451 to 0.1387 as the t_{cl} is increased from 0.10s to 0.15s. System stability condition deteriorates with increase in t_{cl}, which is reflected in the decrease of η. This is in accordance with the results for the classical model case (Table I). The values of η for TCSC placed in different lines of the system are also shown in Table VI (A). It can be seen that the best possible location (corresponding to maximum η) is line 5-7 for both the fault locations. It can be verified from Table II that the best possible locations in the classical model case were the same.

The values of η for increasing t_{cl} for two fault locations (one at a time) of the 68-bus system without TCSC are shown in Table VI (B). It can be seen that η gets decreased with increase in t_{cl}. These results are in accordance with similar results in Table IV, for system with classical model of generators. Also the best possible locations of TCSC (in terms of stability improvement) for these fault locations are shown in Table VI (B). A comparison of these with Table V (A) and (B) shows that the best possible TCSC locations are the same for the two different generator models.

VI. CONCLUSIONS

This paper presents the trajectory sensitivity analysis of a power system with TCSC. We have investigated the influence of firing angle, change of load and generation as well as suitable placement of the TCSC when the system is closer to instability. The trajectory sensitivities are computed numerically. Analysis is carried out with both constant impedance and constant PQ load. It is shown that the constant impedance load modeling produces more optimistic results. It has also been shown that the trajectory sensitivity analysis can be applied to systems with generators modeled by simple classical model as well as by more detailed models of generator like the flux-decay model.

One way to improve the transient stability of a system affected by some contingency like a fault is the application of TCSC. However it is very important to ascertain the changes in stability margin on application of TCSC in a particular location as the stability condition may even deteriorate with TCSC in some locations. This work shows the applicability of TS in making that assessment. The effect of changes in the firing angle on system stability can also be judged using TS. The changes in load and sharing of generators may also influence system stability under contingency. TS can be used to evaluate the corresponding stability margins and thus can help in system operation and planning.

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